

**Math 115**

**Fall 2018**

**Lecture 28**

$a^2 + b^2 = c^2$   
 $y = mx + b$   
 $d = rt$   
?

Feb 19-8:47 AM

## Solving Rational Equations:

- 1) Find the LCD
- 2) Find all excluded Values
- 3) Multiply everything by the LCD to clear denominators.
- 4) Simplify, and Solve the new equation.
- 5) Keep only those Solutions that are not excluded Values.
- ⑥ Don't forget about Solution Set.

Solve  $\frac{5}{x} - 8 = \frac{3}{x}$

LCD =  $x$

E.V. =  $0$

$$x \cdot \frac{5}{x} - x \cdot 8 = x \cdot \frac{3}{x}$$

$$5 - 8x = 3$$

$$-8x = 3 - 5$$

$$-8x = -2$$

$$\Rightarrow x = \frac{-2}{-8}$$

$$\boxed{x = \frac{1}{4}} \Rightarrow \left\{ \frac{1}{4} \right\}$$

Solve  $\frac{5}{x} + 4 = \frac{2}{x}$

LCD =  $x$

E.V. =  $0$

$$x \cdot \frac{5}{x} + x \cdot 4 = x \cdot \frac{2}{x}$$

$$5 + 4x = 2$$

$$\Rightarrow 4x = 2 - 5$$

$$4x = -3$$

$$\boxed{x = \frac{-3}{4}} \Rightarrow \left\{ \frac{-3}{4} \right\}$$

Solve  $\frac{x}{x-4} - 5 = \frac{4}{x-4}$

LCD =  $x-4$

E.V. =  $4$

$$\cancel{(x-4)} \cdot \frac{x}{\cancel{x-4}} - \cancel{(x-4)} \cdot 5 = \cancel{(x-4)} \cdot \frac{4}{\cancel{x-4}}$$

$$x - 5(x-4) = 4$$

$$x - 5x + 20 = 4$$

$$-4x = 4 - 20$$

$$-4x = -16$$

$$x = \frac{-16}{-4} \Rightarrow x = 4$$

No Soln.

$\emptyset$

$\{ \}$

Solve  $\frac{x}{x+5} = 2 - \frac{5}{x+5}$

LCD =  $x+5$

E.V. =  $-5$

$$\cancel{(x+5)} \cdot \frac{x}{\cancel{x+5}} = (x+5) \cdot 2 - \cancel{(x+5)} \cdot \frac{5}{\cancel{x+5}}$$

$$x = 2(x+5) - 5$$

$$x = 2x + 10 - 5$$

$$x - 2x = 5$$

$$-x = 5$$

$$\rightarrow x = \frac{5}{-1}$$

$$\boxed{x = -5}$$

no Soln,  $\emptyset$ ,  $\{ \}$

Solve:  $\frac{3}{x-5} - \frac{2}{x+5} = \frac{20}{x^2-25}$

LCD =  $(x-5)(x+5)$

E.V. =  $5, -5$

$$\cancel{(x-5)}(x+5) \cdot \frac{3}{\cancel{x-5}} - \cancel{(x-5)}(x+5) \cdot \frac{2}{\cancel{x+5}} =$$

$$\cancel{(x-5)}(x+5) \cdot \frac{20}{\cancel{x^2-25}}$$

$$3(x+5) - 2(x-5) = 20$$

$$3x + 15 - 2x + 10 = 20 \rightarrow x = 20 - 25$$

$$x + 25 = 20$$

$$\cancel{x = -5} \text{ See E.V.}$$



Solve  $\frac{2x}{x^2-16} - \frac{2}{x-4} = \frac{4}{x+4}$

LCD =  $(x-4)(x+4)$

E.V. =  $4, -4$

$$\cancel{(x-4)}\cancel{(x+4)} \cdot \frac{2x}{\cancel{x^2-16}} - \cancel{(x-4)}\cancel{(x+4)} \cdot \frac{2}{\cancel{x-4}} = \cancel{(x-4)}\cancel{(x+4)} \cdot \frac{4}{\cancel{x+4}}$$

$2x - 2(x+4) = 4(x-4)$

$\cancel{2x} - \cancel{2x} - 8 = 4x - 16$

$-8 + 16 = 4x$

$\rightarrow 8 = 4x$

$\boxed{2 = x}$

$\{2\}$

Solve:  $\frac{1}{x+4} + \frac{x}{x-4} = \frac{-8}{x^2-16}$

LCD =  $(x+4)(x-4)$

E.V. =  $\pm 4$

$$\cancel{(x+4)}\cancel{(x-4)} \cdot \frac{1}{\cancel{x+4}} + \cancel{(x+4)}\cancel{(x-4)} \cdot \frac{x}{\cancel{x-4}} = \cancel{(x+4)}\cancel{(x-4)} \cdot \frac{-8}{\cancel{x^2-16}}$$

$1(x-4) + x(x+4) = -8$

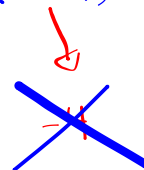
$x-4 + x^2+4x + 8 = 0$

$x^2 + 5x + 4 = 0$

$\rightarrow (x+1)(x+4) = 0$

$\boxed{x = -1}$

$\{-1\}$



Solve  $\frac{2}{x} = \frac{x}{5x-12}$

Hint: Cross-Multiply

$$x \cdot x = 2(5x-12)$$

$$x^2 = 10x - 24$$

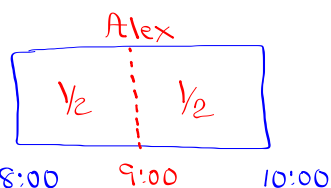
$$x^2 - 10x + 24 = 0$$

$$(x-6)(x-4) = 0$$

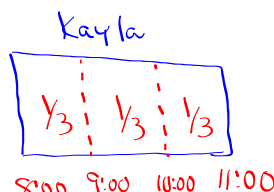
$$\boxed{x=6} \checkmark \quad \boxed{x=4} \checkmark$$

$$\{4, 6\}$$

work problems:



Rate  $\frac{1}{2}$  job/hr



Rate  $\frac{1}{3}$  job/hr.

How long if they work together?

work by Alex + work by Kayla = one complete work

Rate  $\cdot$  Time

Rate  $\cdot$  Time

$$\frac{1}{2} \cdot t + \frac{1}{3} \cdot t = 1 \quad \rightarrow \quad 6 \cdot \frac{t}{2} + 6 \cdot \frac{t}{3} = 6 \cdot 1$$

$$3t + 2t = 6$$

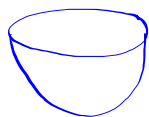
$$\frac{t}{2} + \frac{t}{3} = 1$$

LCD = 6

$$5t = 6$$

$$\boxed{t = 1.2}$$

1.2 hrs



Hot water can fill it up in 5 minutes.

Drain can empty in 10 minutes.

If hot water running, and drain left open, how long does it take to fill up the sink?

work by Water — work by drain = one complete work

$$\frac{1}{5} \cdot t - \frac{1}{10} \cdot t = 1 \quad \frac{t}{5} - \frac{t}{10} = 1$$

LCD = 10

$$10 \cdot \frac{t}{5} - 10 \cdot \frac{t}{10} = 10 \cdot 1$$

$$2t - t = 10$$

$$t = 10$$

10 minutes

Alex can paint a room 3 times as fast as Kayla.

Together, they can paint the room in 21 hrs.  
Find how long for each one if they work alone.

Alex  $\rightarrow x$  hrs  $\rightarrow \frac{1}{x}$  Rate

Kayla  $\rightarrow 3x$  hrs  $\rightarrow \frac{1}{3x}$  Rate

work by Alex + work by Kayla = 1  
Rate  $\cdot$  time Rate  $\cdot$  time

$$\frac{1}{x} \cdot 21 + \frac{1}{3x} \cdot 21 = 1$$

Alex  $\rightarrow 28$  hrs  
Kayla  $\rightarrow 84$  hrs

$$\frac{21}{x} + \frac{21}{3x} = 1$$

$$\frac{21}{x} + \frac{7}{x} = 1$$

$$21 + 7 = x \quad x = 28$$

It takes Luis 9 hrs longer than Iris to do a job.

Together, They can do it in 6 hrs.

How long if they work alone?

Iris  $\rightarrow x$  hrs  $\rightarrow \frac{1}{x}$  rate

Luis  $\rightarrow x+9$  hrs  $\rightarrow \frac{1}{x+9}$  rate

Work by Luis + Work by Iris = 1 comp. wk

Rate  $\cdot$  time      Rate  $\cdot$  time

$$\frac{1}{x+9} \cdot 6 + \frac{1}{x} \cdot 6 = 1$$

$$\frac{6}{x+9} + \frac{6}{x} = 1 \quad \cancel{x(x+9)} \cdot \frac{6}{x+9} + \cancel{x(x+9)} \cdot \frac{6}{x} = \cancel{x(x+9)} \cdot 1$$

$$\text{LCD} = (x+9)x$$

$$\text{E.V.} = -9 \neq 0$$

$$6x + 6(x+9) = x^2 + 9x$$

$$6x + 6(x+9) = x^2 + 9x$$

$$6x + 6x + 54 = x^2 + 9x$$

$$12x + 54 = x^2 + 9x$$

$$x^2 + 9x - 12x - 54 = 0$$

$$x^2 - 3x - 54 = 0$$

$$(x-9)(x+6) = 0$$

$$x = 9$$

$$x = -6$$

Iris 9 hrs

Luis 18 hrs

**NO  
Neg.  
time**

Solve

$$\frac{5}{x+5} - \frac{3}{x-3} = \frac{2}{x^2 + 2x - 15}$$

$$\text{LCD} = (x+5)(x-3) \quad \text{E.V.} = -5, 3$$

$$5(x-3) - 3(x+5) = 2$$

$$5x - 15 - 3x - 15 = 2$$

$$2x = 32 \quad \boxed{x=16} \quad \{16\}$$

Solve

$$-4 \begin{cases} 2x + 3y = -5 \end{cases}$$

$$3 \begin{cases} 3x + 4y = -8 \end{cases}$$

$$\begin{cases} -8x - 12y = 20 \end{cases}$$

$$\begin{cases} 9x + 12y = -24 \end{cases}$$

$$3(-4) + 4y = -8$$

$$-12 + 4y = -8$$

$$4y = 4 \quad \boxed{y=1}$$

$$\boxed{x = -4}$$

$$(-4, 1)$$



Solve by quadratic formula:

$$9x^2 = 12x - 4$$

$$9x^2 - 12x + 4 = 0$$

$$a=9 \quad b=-12 \quad c=4$$

$$b^2 - 4ac = (-12)^2 - 4(9)(4) = 144 - 144 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-12) \pm \sqrt{0}}{18} = \frac{12 \pm 0}{18}$$
$$\left\{ \frac{2}{3} \right\} \text{ Repeated Soln.} = \frac{12}{18} = \boxed{\frac{2}{3}}$$